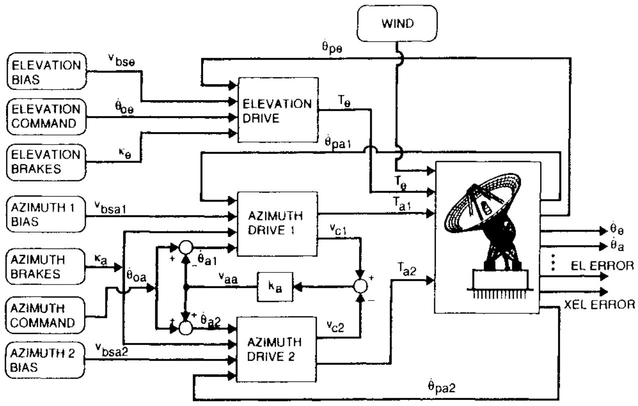

Fig. 2 Rate loop control system of the DSS-13 antenna.

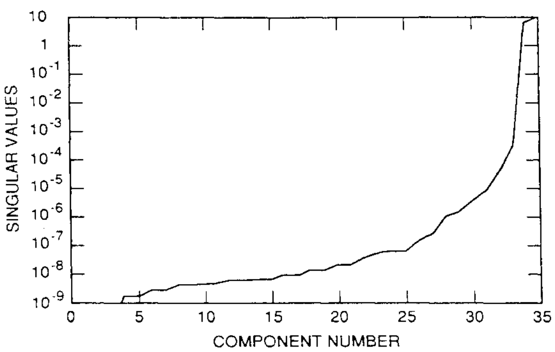

Fig. 3 Singular values of the balanced antigrammians of the rate loop model.

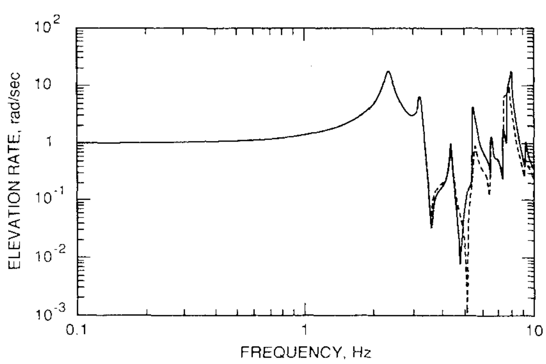

Fig. 4 Magnitude of the elevation rate transfer function (rad/s) for the full and the reduced rate loop model.

Consider the NASA Deep Space Network DSS-13 antenna rate loop model (see Fig. 2; for details of the model see Ref. 2). It consists of the antenna structural model and models of elevation and azimuth drives. The 35-state rate loop model has three poles at zero and is reduced while using balanced antigrammians; their singular values are shown in Fig. 3. By deleting the states with the largest singular values, the model has shrunk from 90 states to 27 states. The reduced model preserves the full-model properties, as illustrated by the frequency-response plots in Fig. 4.

Conclusions

Antigrammians, similarly to grammians, reflect controllability and observability properties of a system. Unlike grammians, antigrammians do exist for systems with integrators, hence making their reduction possible. In this Note the reduction algorithm for systems with integrators is derived. The algorithm is based on balancing the antigrammians and has the same computational effectiveness as the regular balancing procedure. It was applied to the reduction of the NASA Deep Space Network antenna model.[2] As a result, the 90-state model has been reduced to a 27-state model, which preserves the full-model properties.

Acknowledgment

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Minimizing Selective Availability Error on Satellite and Ground Global Positioning System Measurements

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Introduction

THE global positioning system (GPS) will turn on selective availability (SA) encryption, on a regular basis, to degrade the positioning accuracy for users denied access to such encryption. The SA reduces the user positioning accuracy in two ways. First, artificial offsets are added onto the broadcast GPS ephemerides and, second, the GPS clocks that generate the carrier phase and coded signals are dithered. For non-real-time users, the first aspect is of no concern. The effects of the SA clock dithering may in principle be removed by differencing between receivers observing the same GPS satellites. However, this is possible only if the receiver clocks are sufficiently good to keep the time-tags accurate to better than 1 ms. Although most ground receiver clocks can be synchronized to high accuracy, nonsimultaneity as large as 20 ms exists due to the unequal light-time between a given GPS satellite and different receivers. For receivers onboard a user satellite, the nonsimultaneity may be far greater. For instance, the U.S./French ocean topography experiment satellite, Topex/Poseidon,[1] which will be in orbit during 1992–1995, will carry a GPS receiver driven by a free-running crystal clock. Although the clock will be monitored from time to time using onboard real-time navigation, it will not be realigned to GPS clocks so that continuity in carrier phase can be maintained. Hence, the time-tags on the measurements will be drifting away from the correct time. The clock information will be recorded and telemetered back to ground, together with the tracking data.

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When the discrete time-tags are later corrected using the telemetered clock information, the data will be nonsimultaneous with ground receivers, leaving GPS clock (including SA effect) cancellation imperfect. With the planned 1-s sampling for the carrier phase and 10-s sampling for the pseudorange data, the maximum nonsimultaneity from ground receivers is 0.5 and 5 s, respectively, for the two data types.

In the following sections, the characteristics of SA clock dithering are briefly reviewed. A data reduction scheme for the GPS measurements from both satellite and ground receivers is proposed and analyzed. A simulation analysis is performed to demonstrate the effectiveness of the scheme.

Characteristics of SA Effects

Several block II GPS satellites are already in orbit, with the SA signal turned on. Carrier-phase data from ground receivers observing these satellites have shown the fluctuation due to SA signal as high as 30 m. A spectral analysis on a sample of SA signal generated by the classified SA algorithm used by GPS operation indicates that its power spectrum contains only low-frequency components. Earlier analysis[2] on the SA signal extracted from actual carrier-phase data shows similar low-frequency characteristics. In other words, the SA signal behaves as if it had a relatively slowly time-varying frequency offset from an otherwise undithered clock. This allows a low-order interpolation process to be performed on the tracking data so that data received at a user satellite and all ground sites can be made simultaneous, i.e., interpolated so that all signals have a common transmitting time.

SA on GPS Measurements Onboard a Satellite

Without loss of generality, we shall adopt Topex as an illustrating example of SA effects on in-flight GPS measurements. The GPS receiver onboard Topex will make carrier-phase measurement at a rate of once every second, and P-code pseudorange at once every 10 s. When later brought back to the central processing site, these data will then be compressed to a lower data rate for orbit determination in the following filtering process. A compressed data rate of one per 5 min has been shown to be adequate[3] for orbit determination of Earth satellites as low as a few hundred kilometers in altitude. At the maximum time-tag offset of 0.5 s for the 1-s carrier-phase data received at Topex, the differential (from a ground receiver free of time-tag error) SA effect is estimated to be about 10 cm. Because of its low-frequency characteristics, this error is highly correlated over the expected data compression interval of 5 min and hence will not average down. With the 10-s sampling of Topex pseudorange data, the maximum time-tag offset is 5 s; the residual error due to the differential SA effects is estimated to be 1 m. These levels of error are one to two orders of magnitude higher than the expected data noise and grossly unacceptable for precision orbit determination.[3]

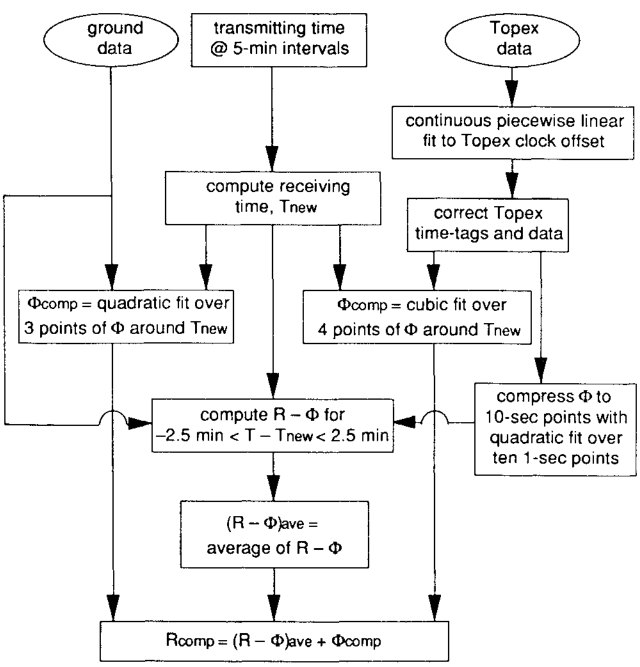

Fig. 1 Flow diagram of GPS data compression schemes for Topex and ground receivers, where R denotes pseudorange and Φ carrier phase.

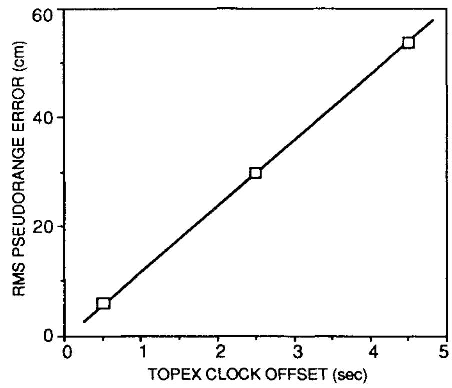

Fig. 2 The rms SA effects on differential pseudorange between Topex and a ground receiver.

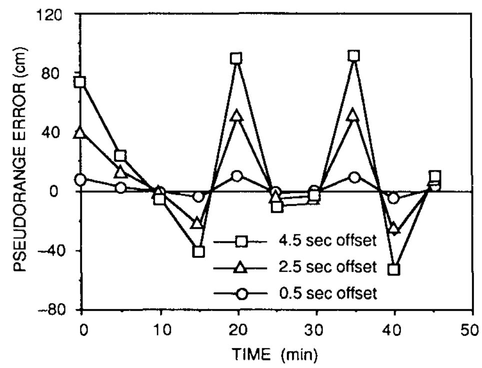

Fig. 3 SA effects on a pass of differential pseudorange between Topex and a ground receiver.

Incomplete Cancellation of SA Effects Due to Light-Time Delay

GPS measurements are commonly tagged by receiving time. On the other hand, SA effects are common only between measurements with the same transmitting time. Such nonsimultaneity in transmitting time will result in incomplete cancellation of GPS clock errors. The difference in light-time between different receivers observing a given GPS satellite can be as large as 20 ms; the difference in SA effect is estimated to be a few millimeters for both pseudorange and carrier-phase data. Although such error is negligible for the pseudorange (with an expected data noise of 5–10 cm), it is significant to carrier-phase data. This error can also be reduced by interpolation to common transmitting time. Shifting the time-tags from common receiving time to common transmitting time, in fact, trades the errors due to the GPS clocks (with SA) for those due to the far better receiver clocks (the receiving times are now nonsimultaneous, but they will have far smaller errors than the SA effects).

Data Compression Scheme

P-code pseudorange data noise of a well-designed GPS receiver is of the order of a few centimeters when averaged to 5-min points.[4] On the other hand, carrier phase is a precise

Table 1 The rms differential pseudorange error using different interpolation strategies on Topex carrier phase with 0.5-s time offset

Interpolation polynomial		
Linear	Quadratic	Cubic
74.5 cm	0.4 mm	<0.1 mm

Table 2 The rms differential pseudorange error using different interpolation strategies on ground carrier phase with 20-ms time offset

Interpolation polynomial		
Linear	Quadratic	Cubic
7.0 mm	<0.1 mm	<0.1 mm

measurement of range change, with a data noise typically two orders of magnitude lower. To keep the pseudorange data noise low, a compression scheme should take advantage of smoothing over the entire 5-min integration (averaging) time. On the other hand, for proper smoothing the signature of the satellite dynamics should be removed with a reasonably good model, which is not always conveniently available. A judicious compression scheme that removes the dynamics without requiring a dynamic model is recommended in the following.

The gross differential SA effects without correcting for the time-tag offset were first assessed. The raw data without including satellite dynamics were generated and then decimated to 5-min data points. Differenced data were formed between Topex and ground receivers observing the same GPS satellites. The rms effects of 18 passes of differenced data observing 10 different GPS satellites having the longest common view periods, over the entire 2-h orbital cycle, were computed and plotted in Fig. 2 for three different Topex clock offsets: 0.5, 2.5, and 4.5 s. The residual effects on carrier phase for the three Topex clock offsets are the same as those on pseudorange at 0.5-s offset since Topex carrier phase is sampled at 1-s intervals. Without proper correction for the time-tag offsets, the residual SA effects on the differenced data are indeed large: 6 cm for carrier phase and about 12 cm for each second of Topex clock offset for pseudorange, up to 60 cm at the maximum offset of 5 s. Note that these are the rms values over the 18 passes; at times the errors are a factor of 3 higher. The residual SA effects without time-tag correction for a typical pass of differential pseudorange between Topex and a ground receiver are shown in Fig. 3.

Next, we investigated the effectiveness of the data compression scheme for reducing the SA effects using different interpolation strategies (linear, quadratic, and cubic). Satellite dynamics were now included in the raw data. The data were compressed and differenced between Topex and ground receivers. These results are then differenced from the "truth" models where the SA effects are turned off. The rms residual errors are summarized in Table 1. A linear interpolation results in a large error (75 cm) due to poor modeling of the satellite dynamics, and thus should not be adopted. With the proposed data compression scheme the effects on both pseudorange and carrier phase are reduced to 0.4 mm using a quadratic interpolation and to <0.1 mm using a cubic interpolation. Although the difference is negligible for pseudorange, cubic interpolation clearly is superior over quadratic for carrier phase, which has a low data noise comparable to the 0.4-mm level. Hence, a cubic interpolation of carrier phase is recommended for Topex data compression. A similar comparison was also made (Table 2) for ground data assuming a 20-ms clock offset due to light-time difference. Because of lower dynamics and small time offset, the error is only 7 mm with a linear interpolation. It reduces to below 0.1 mm with either a quadratic or a cubic interpolation. Hence, a quadratic interpolation of carrier phase is appropriate for ground data compression.

Conclusions

The proposed data compression scheme reduces the selective availability error on satellite and ground differential GPS measurements from 1 m to below 0.1 mm. The residual error will be increased when selective availability of a higher level is turned on. However, the residual error will still be lower than 1 mm even with an increase in the level by an order of magnitude. Although a specific satellite, Topex/Poseidon, has been used as an illustrating example, the scheme is readily applicable to other satellites with similar data-acquisition scenario. The data compression scheme involves only simple algorithms which are computationally efficient. Hence, it can conveniently be incorporated into the data-acquisition software of GPS receivers, where the "raw" data sampling rates are dictated. In general, the proposed scheme applies to non-real-time satellite orbit determination. Real-time applications would require accurate GPS orbits independent of the broadcast ephemerides, and data transmission between receivers for real-time data differencing.

Since carrier-phase data noise is intrinsically low, smoothing over the entire 5-min integration time is not necessary. Instead of removing the satellite dynamics with a good model, a low-order polynomial interpolation over a short time period can be used for the compression of carrier phase. The simulation analysis in the following section indicates that for Topex data at 1-s intervals, a cubic interpolation over four points every 5 min is appropriate even with the strong Topex dynamics. For ground data, a compression scheme with a quadratic interpolation over three 10-s data points every 5 min can be adopted. This is based on the fact that the dynamics are much lower and the corrected time-tag for the ground data will always be less than 0.1 s away from the nearest raw data point, hence a quadratic fit would be appropriate; the corrected time-tag for Topex could be as large as 0.5 s away from the nearest raw data point and a cubic interpolation is recommended to assure a low interpolation error. The following simulation analysis will demonstrate that the differential SA effects between Topex and ground data are small despite different interpolation polynomials used.

The higher data noise in pseudorange discourages the use of any data decimation. However, since the precise carrier phase has identical satellite dynamics as the pseudorange, it can be treated as a dynamics model and subtracted from the pseudorange. Then the dynamics-removed pseudorange can be compressed with a simple averaging over the entire 5-min period. The dynamics is later recovered by adding the compressed carrier phase to the compressed pseudorange. This scheme is called smoothing of pseudorange using carrier phase[5] and has been used in data-acquisition software in some GPS receivers. Since the removal of dynamics using carrier phase also removes the SA effects, the compression through averaging of pseudorange does not introduce any additional interpolation error. Hence, the compressed differential pseudorange will have the same residual SA effects as the compressed differential carrier phase, which are low due to the small (1-s) data intervals. Note that it is important to maintain identical SA effects in pseudorange and carrier-phase data types since they are later removed as common clock error in the filtering process (which is comparable to differencing). A flow diagram summarizing the data compression scheme is shown in Fig. 1.

Simulation Analysis

To assess the effectiveness of the proposed data compression scheme in reducing the SA effects, a simulation analysis was carried out. The orbits of Topex and a constellation of 18 GPS satellites in six orbital planes were computed over a 2-h period. Simulated "raw data," at 1-s intervals for Topex carrier phase and 10-s intervals for Topex pseudorange and both data types for six globally distributed ground receivers, were generated. The SA effects for the 18 GPS satellites were simulated and added to the simulated data. All ground receivers' clocks were assumed to be perfect, whereas different levels of Topex clock offsets were considered.

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Global Transformation of Rotation Matrices to Euler Parameters

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Introduction

TWO widely used forms of attitude parameterization are the rotation matrix form and the Euler parameter form. Each form is globally nonsingular and numerically ideal because rotation group algebra in terms of each involves orthogonal transformations. The transformation of one form to the other is of both practical and theoretical interest. The global transformation of Euler parameters to rotation matrices can be expressed globally as a single matrix equation, but the same is not true of the opposite transformation.

The transformation of rotation matrices to Euler parameters was solved by Klumpp[1] and Shepperd,[2] but their transformation algorithms are not truly global. The Klumpp algorithm is not truly global because for certain attitudes it produces indeterminate forms that must be resolved by additional steps. The Shepperd algorithm is not truly global because it uses different nonglobal transformations for different regions of the attitude state space.

This Note presents the first truly global algorithm for transforming rotation matrices to Euler parameters. Although it has no apparent computational or numerical advantage over the known algorithms, it provides insight into the relationship between the rotation matrix form and the Euler parameter form. It makes use of the singular-value decomposition, a numerically ideal algorithm involving orthogonal transformations. The subsequent sections review attitude parameterization, provide an analytical framework, and finally present the new transformation algorithm.

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Attitude Parameterization

According to Euler's theorem, the attitude of a rigid body can be reached from any arbitrary reference attitude by a rotation about an axis referred to as the Euler axis or eigenaxis. Suppose that two Cartesian coordinate frames are specified: an arbitrary reference frame, and a body frame that is fixed with respect to the rigid body. The reference attitude is defined such that the body coordinate frame is aligned with the reference coordinate frame. Let $a \in \mathbb{R}^3$ represent the identical body and reference coordinates of a unit vector aligned with the eigenaxis, and let $\phi \in \mathbb{R}$ represent the angle of rotation about the eigenaxis, defined in a right-hand sense. Let $R \in \mathbb{R}^{3\times 3}$ and $\beta \in \mathbb{R}^4$ represent the rotation matrix and the Euler parameters, respectively, corresponding to the attitude of the body frame relative to the reference frame. Also, let $\epsilon \in \mathbb{R}^3$ represent the first three Euler parameters, and let $\eta \in \mathbb{R}$ represent the fourth.

The rows of R are the reference coordinates of unit vectors aligned with the corresponding body axes; the columns of R are the body coordinates of unit vectors aligned with the corresponding reference axes. Premultiplication by R transforms the reference coordinates of a vector to the body coordinates of the same vector; premultiplication by R^T does the opposite. The rotation matrix R is expressed in terms of the eigenaxis coordinates a and the rotation angle ϕ as

$$R = \exp(-\phi a \times) \tag{1}$$

$$= (\cos\phi)I + (1-\cos\phi)aa^T - (\sin\phi)a\times \tag{2}$$

where the skew-symmetric cross-product operator is defined for an arbitrary three-component variable as follows:

$$\kappa \equiv \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix} \rightarrow \kappa\times \equiv \begin{bmatrix} 0 & -\kappa_3 & \kappa_2 \\ \kappa_3 & 0 & -\kappa_1 \\ -\kappa_2 & \kappa_1 & 0 \end{bmatrix} \tag{3}$$

Because $\kappa\times\kappa = 0$, $\kappa\times$ is singular, and κ is in its null space. Because the reference frame and the body frame are both orthogonal, so is the rotation matrix R; hence $R^T R = RR^T = I$, $R^{-1} = R^T$, and all three singular values of R are 1.

The elements of β are expressed by definition in terms of the eigenaxis coordinates a and the rotation angle ϕ as

$$\epsilon = \sin(\phi/2)a \tag{4}$$

$$\eta = \cos(\phi/2) \tag{5}$$

The Euler parameters (which are equivalent to the coefficients of a unit quaternion) have unit norm by definition, hence $\|\beta\|^2 = \beta^T\beta = \epsilon^T\epsilon + \eta^2 = 1$. The Euler parameters do not uniquely parameterize attitude because if the signs of all four parameters are changed they still correspond the same physical attitude (they correspond mathematically to an odd number of complete revolutions about the eigenaxis).

Transformation of Euler parameters to rotation matrices is expressed as[3,4]

$$R = (\eta^2 - \epsilon^T\epsilon)I + 2\epsilon\epsilon^T - 2\eta\epsilon\times \tag{6}$$

Analysis

Singular-value decomposition[5,6] is the factorization of any matrix into a product of the form USV^T, where U and V are